TIME SUPPLEMENT

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1. What Are Instants and Durations?

A duration is an amount of time. The duration of earth's existence is about five billion years; the duration of a flash of lightning is 0.0002 seconds. Years and seconds are not durations; they are measures of durations. The second is the standard unit for the measurement of time in the SI system (the International Systems of Units, that is, Le Système International d'Unités). In informal conversation, an instant is a very short duration. In physics, however, an instant is instantaneous; it is not a finite duration but rather a "point" in time, or "a time." The day begins at the instant called "midnight." It is an interesting question whether a finite duration of a real event is always a linear continuum of instants, and, if so, how we know this.

2. What Is an Event?

In ordinary discourse, an event is a happening lasting a finite duration during which some object changes its properties. For example, this morning's event of buttering the toast is the toast's changing from unbuttered to buttered.

Without mentioning objects and properties, an event might instead be defined simply as whatever is temporally before or after anything else. In physics, events are considered to be more basic than objects and their properties. But if we do treat events in terms of objects and properties, we might treat the buttering event as involving the toast object having changed from not having the property of containing butter at a certain time this morning at a certain location to its having the property of containing butter at that location a few seconds later.

In ordinary discourse, an event has more than an infinitesimal duration, but in the technical discourse of physics, all events are composed of point events, events with zero duration and taking up zero volume of space (that is, being extensionless). Also, a actual point event is considered by physicists to be a spacetime point's having some property other than those it has just by being a location in spacetime. By being so brief and taking place in such a restricted volume, these point events are very idealized, but they are very useful, and each location in spacetime is marked by an actual or *possible* point event occurring there. For actual point events, the point event can be thought of as the point's having some property for an instant. Notice that no change is mentioned here, nor is a physical object that has those properties. Point events are what all objects and events are made of, and spacetime points are what have the properties.

A mathematical space is a collection of points, and the points might represent anything, for example dollars. But the points of a real space such as spacetime are actual and possible point events, and the points of a real space that is time are instants.

These metaphysical assumptions of modern science are not part of common sense, the shared background beliefs of most people. They also are not acceptable metaphysical assumptions for many philosophers. In 1936, in order to avoid point events, Bertrand Russell and A. N. Whitehead developed a theory of time based on the assumption that all events in spacetime have a finite, non-zero duration. However, they had to assume that any finite part of an event is an event, and this assumption is no closer to common sense than the physicist's assumption that all events are composed of point events. The encyclopedia article on Zeno's Paradoxes me ntions that Michael Dummett and Frank Arntzenius have continued in the 21st century to develop Russell's and Whitehead's ideas about events having a finite, non-zero duration.

It is an open question in philosophy as to whether the passage of time is a feature of the world to be explained by noting how events change, such as their changing from being present to being past. Many philosophers believe it is improper to consider an event to be something that can change.

For a more detailed discussion of what an event is, see the article Events.

3. What Is a Reference Frame?

A reference frame for a space is a coordinate system, namely a standard point of view or a perspective for making observations, measurements and judgments that assigns unique values to each point of space.

Choosing a good reference frame can make a situation much easier to describe. If you are trying to describe the motion of a car down a straight highway, you would not want to choose a reference frame that is fixed to a spinning carousel. Instead, choose a reference frame fixed to the highway. The motion of a planet is very complex as seen from earth over many months. However, the motion is very simple in a frame of reference at rest relative to the sun. *Inertial frames* are very special reference frames, as we shall see below.

A reference frame is often specified by selecting a solid object that doesn't change its size and by saying that the reference frame is fixed to the object. We might select a reference frame fixed to the Rock of Gibraltar. Another object is said to be at rest in the reference frame if it remains at a constant distance in a fixed direction from the reference body used to define the frame. For example, your house is at rest in a reference frame fixed to the Rock of Gibraltar [not counting your house's vibrating when a truck drives by, nor the house's speed due to plate tectonics]. When we say the sun rose this morning, we are implicitly choosing a reference frame fixed to the earth's surface. The sun is not at rest in this reference frame.

The reference frame or coordinate system must specify locations, and this is normally done by assigning numbers to points of space. In a flat (that is, Euclidean) three-dimensional space, the analyst needs to specify four distinct points on the reference body, or four objects mutually at rest somewhere in the frame. One point is the origin, and the other three can be used to define three independent, perpendicular axes, the familiar x, y and z directions, assuming a Cartesian (that is, flat or rectangular) coordinate system were to be used. Two point objects are at the same place if they have the same x-value, the same y-value and the same z-value. To keep track of events, you will also need a time axis, a "t" axis, and so you will expand your three-dimensional mathematical space to a four-dimensional mathematical space. Two point events are simultaneous if they occur at the same place and also at the same time. In this way, the analyst is placing a four-dimensional coordinate system on the space and time. The coordinates could have been letters instead of numbers, but numbers are the best choice because we want to use them for measurement, not just for naming places.

The fact that physical spacetime has curvature implies that no single rigid (or Cartesian) coordinate system is capable of covering the entire spacetime. To cover all of spacetime in that case, we must make do with covering different regions of spacetime with different coordinate patches that are “knitted together” where one patch meets another. No single coordinate system can cover the surface of a sphere without creating a singularity, but the sphere can be covered by patching together two coordinate systems. Nevertheless if we can live with non-rigid curvilinear coordinates, then any curved spacetime can be covered with a global four-dimensional coordinate system in which every point being uniquely identified with a set of four numbers in a continuous way. That is, we use a curved coordinate system on curved spacetime.

That we use four numbers per point indicates the space is four-dimensional. And in creating coordinate systems for spaces, the usual assumption is that we should supply n independent numbers to specify a place in an n-dimensional space, where n is an integer. This is usual but not required; instead we could exploit the idea that there are space-filling curves which permit a single continuous curve to completely fill, and thus coordinatize, a region of dimension higher than one, e.g., a plane. For this reason (namely, that each point in n-dimensional space doesn’t always need n numbers to name the point), the contemporary definition of “dimension” is rather exotic.

4. What Is an Inertial Frame?

An inertial frame is either a non-accelerating frame, or, less generally, a reference frame in which Newton’s laws of motion hold. Any spacetime obeying the laws of Special Relativity can have an inertial frame across the whole universe. In General Relativity any very small region of spacetime can have an inertial frame.

Suppose you’ve pre-selected your frame. How do you tell if you are in an inertial frame? The answer is that you check that objects accelerate only when acted on by forces. That is, you check that any object’s acceleration is zero if no net force acts on the object. If no unbalanced external forces are acting on a moving object, then the object moves in a straight line. It doesn’t curve; it coasts. And it travels equal distances in equal amounts of time.

Any frame of reference moving at constant velocity relative to an inertial frame is also an inertial frame. A reference frame spinning relative to an inertial frame is never an inertial frame.

Einstein's theory of special relativity is intended to apply only to inertial frames. According to the theory, the speed of light in a vacuum is the same when observed from any inertial frame of reference. Unlike the speed of a spaceship, the speed of light in a vacuum isn't affected by which inertial reference frame is used for the measurement. If you have two relatively stationary, synchronized clocks in an inertial frame, then they will read the same time, but if one moves relative to the other, then they will get out of synchrony. This loss of synchrony due to relative motion is called "time dilation."

The presence of gravitation normally destroys any possibility of finding a perfect inertial frame. Nevertheless, any spacetime obeying the general theory of relativity and thus accounting for gravitation will be locally Minkowskian in the sense that any infinitesimal region of spacetime is an inertial frame obeying the principles of special relativity.

5. What Is Spacetime?

Spacetime is where events are located, or, depending on your theory of spacetime, it's all possible events. Spacetime is usually represented as the model of a four-dimensional mathematical space, one of whose dimensions represents time. The four dimensions include the time dimension of before-after and the three ordinary space dimensions of, say, up-down, left-right, and forward-backward.

More technically, spacetime is the intended model of the general theory of relativity. This requires it to be a differentiable space in which physical objects obey the equations of motion of the theory. Minkowski space (that is, Minkowski spacetime) is the model of *special* relativity. It's a certain 4-dimensional real vector space. General relativity theory requires that spacetime be locally like Minkowski spacetime.

Hermann Minkowski, in 1908, was the first person to say that spacetime is fundamental and that space and time are just aspects of spacetime. Minkowski meant it is fundamental in the sense that the spacetime interval between any two events is intrinsic to spacetime and does not vary with the reference frame, unlike a spatial distance or temporal duration.

Spacetime is a continuum in which we can define points and straight lines. However, these points and lines do not satisfy the principles of Euclidean geometry when matter is present.

Einstein showed that the presence of matter affects geometry by warping space and time. Einstein's principal equation in his general theory of relativity implies that the curvature of spacetime is directly proportional to the density of mass in the spacetime. That is, Einstein says the structure of spacetime changes as matter moves because the gravitational field from matter actually curves spacetime. Black holes are a sign of radical curvature. The earth's curving of spacetime is very slight but still significant enough that it must be accounted for when synchronizing two Global Positioning Satellites.

There have been serious attempts over the last few decades to construct theories of physics in which spacetime is a product of more basic entities. The primary aim of these new theories is to unify relativity with quantum theory. So far these theories have not stood up to any empirical observations or experiments that could show them to be superior to the presently accepted theories. So, for the present the concept of spacetime remains fundamental.

The metaphysical question of whether spacetime is a substantial object or a relationship among events, or neither, is considered in the discussion of the relational theory of time.

6. What Is a Minkowski Diagram?

A spacetime diagram is a representation of the point-events of spacetime. In a Minkowski spacetime diagram, a rectangular coordinate system is used, Einstein's Special Theory of Relativity holds, normally the time axis is vertical, one or two of the spatial axes are suppressed, and an object's inertial motion (coasting) produces events in a straight line.

..The directed arrows represent the path of light rays from the flash. In a Minkowski diagram, a physical object, such as an electron or a person's body, is not represented as occupying a point but as occupying a line containing all the spacetime points at which it exists. The line, which usually isn't straight, is called the worldline of the object. In the above diagram, Einstein's worldline is a vertical line. If an object's worldline intersects or meets another object's worldline, then the two objects have collided. The units along the vertical time axis are customarily chosen to be the product of time and the speed of light so that "worldlines" of light rays make a forty-five degree angles with each axis. The set of all light speed world lines going through an event defines the *light cones* of that event: the past light cone and the future light cone.

Inertial motion produces a straight worldline, and accelerated motion produces a curved worldline. If at some time Einstein were to jump on a train moving by at constant speed, then his worldline would, from that time upward, tilt away from the vertical and form some angle less than 45 degrees with the time axis. Events on the same horizontal line of the Minkowski diagram are simultaneous in that reference frame. A moving observer is added to this diagram to produce the diagram below in the discussion about the relativity of simultaneity. In a coordinate system attached to the Sun, the worldline of the Earth's orbit would be a helix.

Not all spacetimes can be given Minkowski diagrams, but any spacetime satisfying Einstein's Special Theory of Relativity can. Minkowski diagrams are diagrams of a Minkowski space, which is a spacetime satisfying the Special Theory of Relativity. This theory falsely presupposes that physical processes, such as gravitational processes, have no effect on the structure of spacetime. When attention needs to be given to the real effect of these processes on the structure of spacetime, that is, when general relativity needs to be used, then Minkowski diagrams become inappropriate for spacetime. General relativity assumes that the geometry of spacetime is locally Minkowskian but not globally. That is, spacetime is locally flat in the sense that in any very small region one always finds spacetime to be 4-D Minkowskian (but not 4-D Euclidean). Special relativity holds in infinitesimally small region of spacetime that satisfies general relativity, and so any such region can be fitted with an inertial reference frame. When we say spacetime is "really curved" and not flat, we mean it really deviates from 4-D Minkowskian geometry.

7. What Are the Metric and the Interval?

A space is simply a collection of points. How far is it from one point to some different point? The metric is the answer to this question; the "spacetime interval" is not. A metric on a space provides a definition of distance (or length) by giving a function from each pair of nearby points to a real number. In Euclidean space, the distance between two points is the length of the straight line connecting them. This length is traditionally defined in terms of coordinates using the Pythagorean Theorem.

Points are located by being assigned a coordinate. For doing science we want the coordinate to be a real number, not, say, a letter of the alphabet. A coordinate for a point in two-dimensional space requires two numbers; a coordinate for a point in n-dimensional space requires n numbers. Time, being one-dimensional, requires a single number. Time is considered a one-dimensional space mathematically, and the metric of time is normally chosen to be the absolute value of the numerical difference between the coordinates of the two points. For example, the duration between 5 AM and 8 AM is three hours, assuming the times are for the same day.

In a 2-dimensional space, the metric is more complicated; the distance between the point (x’,y’), with Cartesian coordinates x’ and y’, and the point (x,y), with coordinates x and y, is defined to be the square root of $(x' - x)^2 + (y' - y)^2$ when the space is flat, that is, Euclidean. If the space is not flat, then a more sophisticated definition of the metric is required. Note the application of the Pythagorean Theorem for Euclidean space.

Our intuitive idea of what a distance is tells us that, however we define distance for a space, we want it to have certain distance-like properties. For example, letting d(p,q) stand for the distance between any two points p and q in the space, the following four conditions must be satisfied:

1. d(p,p) = 0, and d(p,q) is greater than or equal to 0
2. If d(p,q) = 0, then p = q
3. d(p,q) = d(q,p)
4. d(p,q) + d(q,r) is greater than or equal to d(p,r)

Notice that there is no mention of the path the distance is taken across; all the attention is on the point pairs themselves.

Do these conditions capture your idea of distance? If you were to check, you’d find that the 2-D metric defined above, namely the square root of $(x' - x)^2 + (y' - y)^2$, does satisfy these four conditions. In 3-D Euclidean space, the metric that is defined to be the square root of $(x' - x)^2 + (y' - y)^2 + (z' - z)^2$ works very well.

Consider the 4-D mathematical space that is used to represent the spacetime obeying the laws of special relativity theory, namely Minkowski spacetime. What’s an appropriate metric for this space? Well, if we were just interested in the space part of this spacetime, then the above 3-D Euclidean metric is fine. But we’ve asked a delicate question because the fourth dimension of this mathematical space is really a time dimension and not a space dimension. Here is the so-called *Lorentzian metric* or *Minkowski metric* for any pair of point events at (x’,y’,z’,t’) and (x,y,z,t) in Minkowskian 4-D spacetime:

$$\Delta s^2 = -(x' - x)^2 - (y' - y)^2 - (z' - z)^2 + c^2(t' - t)^2$$

Δs is called the *interval* of Minkowski spacetime. The interval corresponds to what clocks measure between a pair of timelike events [that is, between a pair of events separated enough in time that one could have had a causal effect on the other] and what rulers measure between a pair of spacelike events. One other happy feature of this metric is that the value of the interval is unaffected by changing to a new reference frame provided the new reference frame is not accelerating relative to the first. Changing from a first frame to a new, unaccelerated reference frame on the spacetime will change the values of all the coordinates of the points of the spacetime, but some relations between all pairs of points won’t be affected, namely the intervals between pairs of points. Take any two observers who use a reference frame in which they, themselves are fixed, and assume they are moving at constant velocity relative to each other. Now consider some single event with a finite duration. The two observers won’t agree on how long the event lasts, but they will always agree on the interval between the beginning and end of the event.

The interval of spacetime is complicated because its square can be negative, unlike with the space intervals we’ve discussed so far. If Δs^2 is negative, the two points have a space-like separation, meaning these events have a greater separation in space than they do in time. If Δs^2 is positive, then the two have a time-like separation, meaning enough time has passed that one event could have had a causal effect on the other. If Δs^2 is zero, the two events might be identical, or they might have occurred millions of miles apart. In ordinary space, if the space interval between two events is zero, then the two events happened at the same time and place, but in spacetime, if the spacetime interval between two events is zero, this means only that there could be a light ray connecting them. All the events that have a zero spacetime interval from some event e constitute e’s two *light cones* because they have the shape of cones when represented in a Minkowski diagram, one cone for events in e’s future and one cone for events in e’s past. It is because the spacetime interval between two events can be zero even when the events are far apart in distance that the term “interval” is very unlike what we normally mean by the term “distance.”

Because true metrics are always positive, the Lorentzian metric that we used above for Minskowski spacetime is not a true metric, nor even a pseudometric; but it is customary for physicists to refer to it loosely as a “metric” because Δs retains enough other features of distance.

The interval in spacetime is an intrinsic feature of spacetime because it does not vary with our choice of the coordinate system. The duration of an event is not intrinsic, nor are the length or velocity of an object. In the Euclidean plane of 2-D space, length is intrinsic. The metric determines the geometry of spacetime, and this geometry is an intrinsic feature of the spacetime. Adding space and time dependence to each term of the Lorentzian metric produces the metric for general relativity.